EXAMPLE 1

HILLIARD Electronics produces specially coded chips for laser surgery in 256MB and 512MB (MB stands for megabyte; where one megabyte is roughly equal to one million characters of information). It takes 13 hours to produce a 256MB chip and 16 hours to produce a 512MB chip. The monthly production capacity of the plant is 1200 hours. The firm's Sales Manager estimates that maximum monthly sales of 256MB and 512MB chips are 50 and 60 respectively.

The company has the following goals (ranked in order from most important to least important);

1. Fill an order from best customer for thirty 256MB chips and thirty five 512MB chips.
2. Provide sufficient chips to at least equal the sales estimates set by Sales Manager.
3. Avoid under-utilization of the production capacity.

Formulate the problem as a Goal Programming problem.

(a) Use graphical procedure and solve. What are best "satisficing" values of X1 and X2; where X1= No of 256MB chips, X2 = No of 512MB chips.
(b) Convert Goal program to linear program format. Use LINGO and find solution.
(c) Interpret solution and give your comments.

{ Hints:

Objective Function:

Goal Programming has more than one objective functions:

Graphical Method

Draw constraints given in problem statement on graph and identify the feasible region.
Find extreme points
Now evaluate the extreme points to evaluate the three goals:

Goal 1 : Produce a minimum 30 of 256MB chips; minimum 35 512MB chips

X_256 >= 30
X_512 >= 35

Goal 2 : Produce maximum 50 of 256MB chips; maximum 60 512MB chips

X_256 >= 50
X_512 >= 60

Goal 3 : Production capacity of 1200 Hours should all be used (13 * X_256 + 16 * X_512 >= 1200)

LP Model (convert the goals using d^+ and d^- variables for each constraint with following goals.

Goal 1 : Produce a minimum 30 of 256MB chips; minimum 35 512MB chips

X_256 >= 30
X_512 >= 35

Goal 2 : Produce maximum 50 of 256MB chips; maximum 60 512MB chips

X_256 >= 50
X_512 >= 60

Goal 3 : Production capacity of 1200 Hours should all be used

13 * X_256 + 16 * X_512 >= 1200

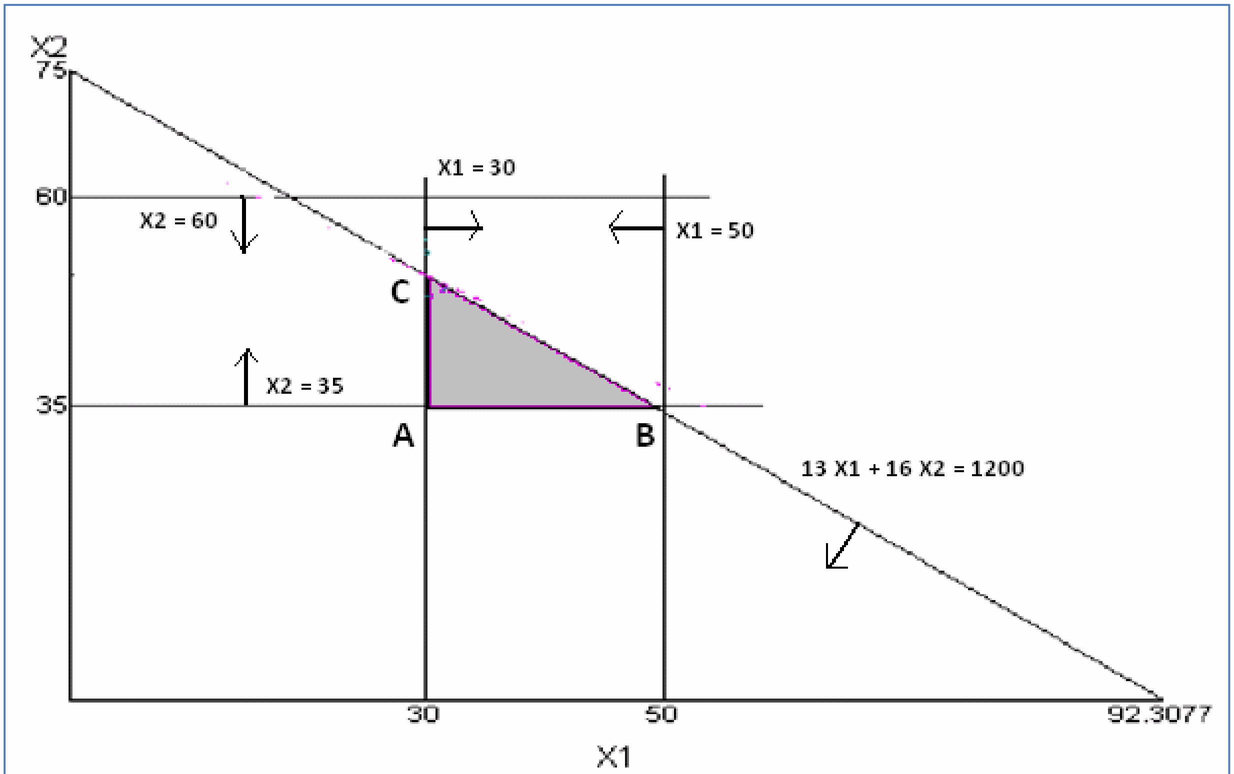
Now solve three LP problems one by one starting from Goal1 up to Goal 3.

Solution:

The region bounded by the constraints for feasibility;

i) Minimum sales of X_256 and X_512 chips;
ii) Upper Limit on sales estimates; X_256 <=50, X_512 <= 60;
iii) Maximum hours of 1200; → 13 X_256 + 16 X_512 <= 1200

Is shown below;



Evaluation of corner point values with regard to Goals;

	Point A : (30,35)	**Point B : (49.23, 35)**	**Point C : (30,50.625)**
Goal 1 (X1>=30, X2>= 35)	Excellent	Excellent	Excellent
Goal 2 (X1>= 50, X2 >= 60)	X1 = -20, X2 = 25	X1 = 0, X2 = -25	X1 = -20, X2 = -9.375
Goal 3 (13 X1 + 16 X2 >= 1200)	250hours left unspent	All hours spent	all hours spent

Point B comes as close to the goals

Linear Programming Solution

Define positive and negative deviation variables for all the goals

For Goal 1 (X1>=30, X2>= 35)

$X1 - 30 = D_1^+ - D_1^-$, $X2 - 35 = D_2^+ - D_2^-$

For Goal 2 (X1>= 50, X2 >= 60)

$X1 - 50 = D_3^+ - D_3^-$, $X2 - 60 = D_4^+ - D_4^-$

For Goal 3 (13 X1 + 16 X2 >= 1200)

$13\ X1 + 16\ X2 - 1200 = D_5^+ - D_5^-$,

Priority 1 → Goal 1

Minimum Sales of X1>=30, X2>=35 must be met. So, minimize negative deviations

LINGO Model - LP

```
MIN = D1N + D2N;

[AVAILABLE_HOURS]        13 * X1 + 16 * X2 <= 1200;

[MIN_X1_SALES]              X1 - 30              = D1P - D1N;
[MIN_X2_SALES]              X2 - 35              = D2P - D2N;
[MAX_X1_SALES]              X1 - 50              = D3P - D3N;
[MAX_X2_SALES]              X2 - 60              = D4P - D4N;
[ALL_HOURS_TO_BE_USED]   13 * X1 + 16 * X2 - 1200 = D5P - D5N;

@GIN(X1);
@GIN(X2);
```

Solution of LP model;

```
Global optimal solution found at iteration:          2
  Objective value:                              0.000000

                     Variable           Value        Reduced Cost
                          D1N        0.000000            0.000000
                          D2N        0.000000            0.000000
                           X1        30.00000           -1.000000
                           X2        35.00000           -1.000000
                          D1P        0.000000            1.000000
                          D2P        0.000000            1.000000
                          D3P        0.000000            0.000000
                          D3N        20.00000            0.000000
                          D4P        0.000000            0.000000
                          D4N        25.00000            0.000000
                          D5P        0.000000            0.000000
                          D5N        250.0000            0.000000
```

With D1N=0, D2N = 0 to be added to constraint set, now introduce Priority Goal 2 objectives for next LP model;

LINGO Model - LP

```
MIN = D3N + D4N;

[AVAILABLE_HOURS]        13 * X1 + 16 * X2 <= 1200;

[MIN_X1_SALES]                X1 - 30             = D1P - D1N;
[MIN_X2_SALES]                X2 - 35             = D2P - D2N;
[MAX_X1_SALES]                X1 - 50             = D3P - D3N;
[MAX_X2_SALES]                X2 - 60             = D4P - D4N;
[ALL_HOURS_TO_BE_USED]   13 * X1 + 16 * X2 - 1200 = D5P - D5N;
                                             D1N = 0;
                                             D2N = 0;
@GIN(X1);
@GIN(X2);
```

Solution LP model for Goal 2 priorities is;

```
Global optimal solution found at iteration:          0
  Objective value:                              26.00000

                     Variable           Value        Reduced Cost
                          D3N        1.000000            0.000000
                          D4N        25.00000            0.000000
                           X1        49.00000           -1.000000
                           X2        35.00000           -1.000000
                          D1P        19.00000            0.000000
```

D1N	0.000000	0.000000
D2P	0.000000	0.000000
D2N	0.000000	0.000000
D3P	0.000000	1.000000
D4P	0.000000	1.000000
D5P	0.000000	0.000000
D5N	3.000000	0.000000

Now add D3N = 1, and, D4N = 25 in constraint set, and minimize D5N to achieve Goal 3 objectives;

```
LINGO Model - LP
MIN = D5N;

[AVAILABLE_HOURS]        13 * X1 + 16 * X2 <= 1200;

[MIN_X1_SALES]                X1 - 30             = D1P - D1N;
[MIN_X2_SALES]                X2 - 35             = D2P - D2N;
[MAX_X1_SALES]                X1 - 50             = D3P - D3N;
[MAX_X2_SALES]                X2 - 60             = D4P - D4N;
[ALL_HOURS_TO_BE_USED]   13 * X1 + 16 * X2 - 1200 = D5P - D5N;
                                             D1N = 0;
                                             D2N = 0;
                                             D3N = 1;
                                             D4N = 25;
@GIN(X1);
@GIN(X2);
```

Solution of Goal 3 objective function;

```
Global optimal solution found at iteration:          0
  Objective value:                            3.000000
```

Variable	Value	Reduced Cost
D5N	3.000000	0.000000
X1	49.00000	-13.00000
X2	35.00000	-16.00000
D1P	19.00000	0.000000
D1N	0.000000	0.000000
D2P	0.000000	0.000000
D2N	0.000000	0.000000
D3P	0.000000	0.000000
D3N	1.000000	0.000000
D4P	0.000000	0.000000
D4N	25.00000	0.000000
D5P	0.000000	1.000000

FINAL SOLUTION: → X1 = 49, X2 = 35 (This is point B on Graph)

EXAMPLE 2 (ASSIGNMENT PROBLEM)

The personnel director of Dollar Finance Corp. must assign three recently hired college graduates to three regional offices. The three new loan officers are equally well qualified, so the decision will be based on the costs of relocation the graduates' families. Cost data are presented in the following table

OFFICER	OFFICE		
	OMAHA	MIAMI	DALLAS
Jones	$800	$1,100	$1,200
Smith	$500	$1,600	$1,300
Wilson	$500	$1,000	$2,300

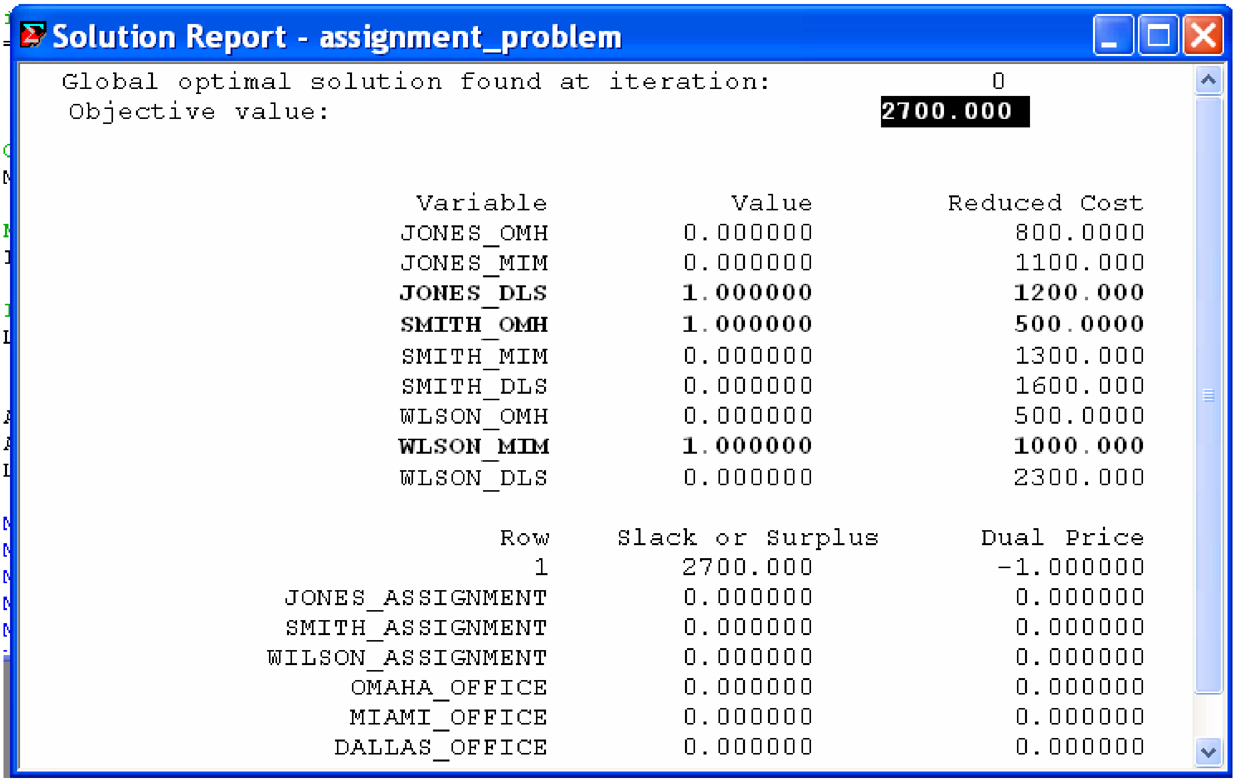
Formulate 0-1 LP model and make cost-effective assignment of officers by using LINGO solver.

Solution { minimize assignment cost}

Let's define binary variables like; JONES_OMH = Jones to be assigned to OMAHA city

LINGO Model - assignment_problem

```
! Minimize total assignment costs;
MIN= 800 * JONES_OMH + 1100 * JONES_MIM + 1200 * JONES_DLS +
     500 * SMITH_OMH + 1300 * SMITH_MIM + 1600 * SMITH_DLS +
     500 * WLSON_OMH + 1000 * WLSON_MIM + 2300 * WLSON_DLS;

! JONES must be assigned to a regional office;
[JONES_ASSIGNMENT] JONES_OMH + JONES_MIM + JONES_DLS = 1;

! SMITH must be assigned to a regional office;
[SMITH_ASSIGNMENT] SMITH_OMH + SMITH_MIM + SMITH_DLS = 1;

! WILSON must be assigned to a regional office;
[WILSON_ASSIGNMENT]WLSON_OMH + WLSON_MIM + WLSON_DLS = 1;


[OMAHA_OFFICE] JONES_OMH + SMITH_OMH + WLSON_OMH = 1;
[MIAMI_OFFICE] JONES_MIM + SMITH_MIM + WLSON_MIM = 1;
[DALLAS_OFFICE]     JONES_DLS + SMITH_DLS + WLSON_DLS = 1;

@BIN(JONES_OMH);
@BIN(JONES_MIM);
@BIN(JONES_DLS);
@BIN(SMITH_OMH);
@BIN(SMITH_MIM);
@BIN(SMITH_DLS);
@BIN(WLSON_OMH);
@BIN(WLSON_MIM);
@BIN(WLSON_DLS);
```



Solution Report - assignment_problem

Global optimal solution found at iteration: 0
Objective value: 2700.000

Variable	Value	Reduced Cost
JONES_OMH	0.000000	800.0000
JONES_MIM	0.000000	1100.000
JONES_DLS	**1.000000**	**1200.000**
SMITH_OMH	**1.000000**	**500.0000**
SMITH_MIM	0.000000	1300.000
SMITH_DLS	0.000000	1600.000
WLSON_OMH	0.000000	500.0000
WLSON_MIM	**1.000000**	**1000.000**
WLSON_DLS	0.000000	2300.000

Row	Slack or Surplus	Dual Price
1	2700.000	-1.000000
JONES_ASSIGNMENT	0.000000	0.000000
SMITH_ASSIGNMENT	0.000000	0.000000
WILSON_ASSIGNMENT	0.000000	0.000000
OMAHA_OFFICE	0.000000	0.000000
MIAMI_OFFICE	0.000000	0.000000
DALLAS_OFFICE	0.000000	0.000000

Optimal Assignment

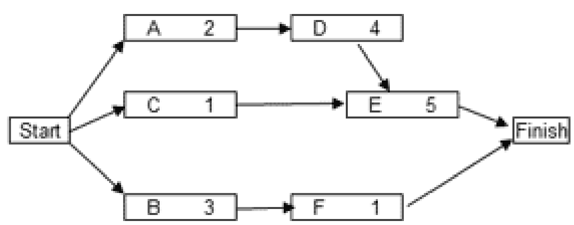
OFFICER	OFFICE		
	OMAHA	**MIAMI**	**DALLAS**
Jones	$800	$1,100	$1,200
Smith	$500	$1,600	$1,300
Wilson	$500	$1,000	$2,300

EXAMPLE 3 (PROJECT PLANNING & SCHEDULING)

Capitol Hill Construction Company (CHCC) must complete its current office building renovation as quickly as possible. The first portion of the project consists of six activities, some of which must be finished before others are started. The activities, their precedence's, and their estimated times are shown in this table:

ACTIVITY	PRECEDENCE	TIME (DAYS)
Prepare financing options (A)	--	2
Prepare preliminary sketches (B)	--	3
Outline specifications (C)	--	1
Prepare drawings (D)	A	4
Write specifications (E)	C and D	5
Run off prints (F)	B	1

This network of tasks can be drawn as shown below.



Formulate and solve CHCC's problem as a linear program. Let *X* represent the earliest completion of an activity where *i* = A, B, C, D, E, F.

Use LINGO and solve. What is the schedule of activities.

Hint:

Obj Fun : Minimize Finish Time
Some of the constraints;
Start_A = 0;
Start_D = Start_A + 2
Activity E has two precedences; activity D and activity C
One of the constraint might be;
Start_E >= Start_C + 1
Continue with the logic, and formulate constraints for all activities.
What is FINISH time….

Linear Programming Model

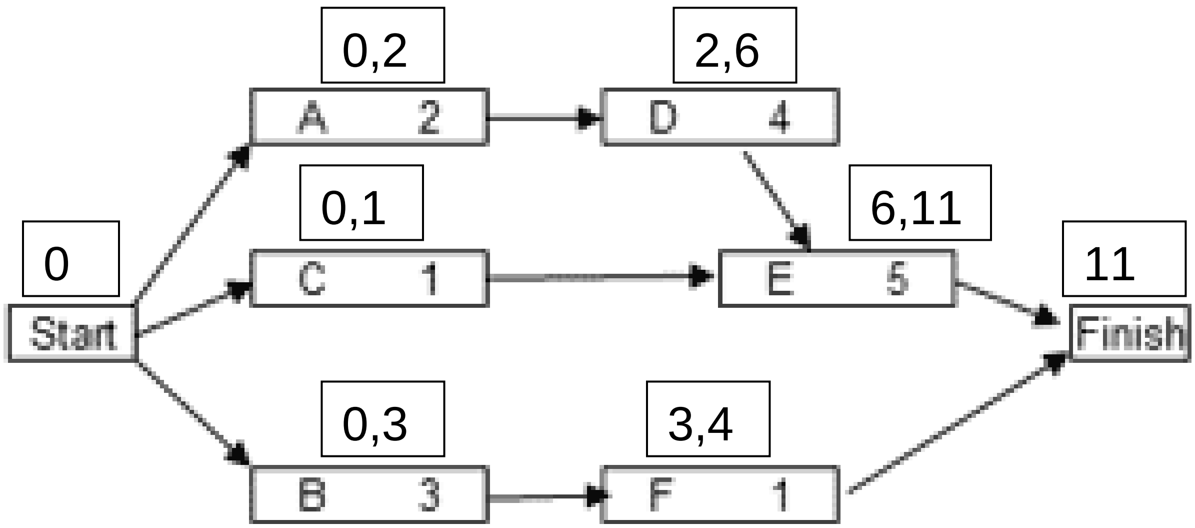
LINGO Model - HW4_CPM_Problem

```
MIN=FINISH_PROJECT;

START = 0;
START_TIME_A = START;
START_TIME_C = START;
START_TIME_B = START;
START_TIME_D = START_TIME_A + 2;
START_TIME_E >= START_TIME_D + 4;
START_TIME_E >= START_TIME_C + 1;
START_TIME_F = START_TIME_B + 3;
FINISH_PROJECT >= START_TIME_D + 4;
FINISH_PROJECT >= START_TIME_E + 5;
FINISH_PROJECT >= START_TIME_F + 1;
COMP_TIME_A = START_TIME_A + 2;
COMP_TIME_B = START_TIME_B + 3;
COMP_TIME_C = START_TIME_C + 1;
COMP_TIME_D = START_TIME_D + 4;
COMP_TIME_E = START_TIME_E + 5;
COMP_TIME_F = START_TIME_F + 1;
```

LP Solution

```
Global optimal solution found at iteration:            4
  Objective value:                              11.00000
```

Variable	Value	Reduced Cost
FINISH_PROJECT	11.00000	0.000000
START	0.000000	0.000000
START_TIME_A	0.000000	0.000000
START_TIME_C	0.000000	0.000000
START_TIME_B	0.000000	0.000000
START_TIME_D	2.000000	0.000000
START_TIME_E	6.000000	0.000000
START_TIME_F	3.000000	0.000000
COMP_TIME_A	2.000000	0.000000
COMP_TIME_B	3.000000	0.000000
COMP_TIME_C	1.000000	0.000000
COMP_TIME_D	6.000000	0.000000
COMP_TIME_E	11.00000	0.000000
COMP_TIME_F	4.000000	0.000000


0,2
A 2
2,6
D 4
0,1
C 1
6,11
E 5
0
Start
11
Finish
0,3
B 3
3,4
F 1